CBCS SCHEME

USN

17MAT31

Third Semester B.E. Degree Examination, July/August 2021 **Engineering Mathematics - III**

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

1 Obtain the Fourier series for the function

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{5^2} + \frac{1}{5^2} + \dots$

(08 Marks)

Find the Fourier series for the function $f(x) = 2x - x^2$ in 0 < x < 3.

(06 Marks)

Obtain the constant term and the first sine and cosine terms of the Fourier for y using the following table:

x:	0	1/	2	3	4	5
y:	4	8	715	7	6	2

(06 Marks)

a. Obtain the Fourier series for the function $f(x) = |\cos x|, \quad \pi < x < \pi$. (08 Marks)

b. Find the Half range cosine series for $f(x) = x(\ell - x)$, $0 \le x \le \ell$. (06 Marks)

c. Express y as a Fourier series upto first harmonic given:

x:	0	$\frac{\pi}{3}$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

(06 Marks)

3 a. If $f(x) = \begin{cases} 1 - x^2, & |x| < 0 \\ 0, & |x| \ge 1 \end{cases}$

Find the Fourier transform of f(x) and hence find the value of $\int_{0}^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) dx$

(08 Marks)

Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_{0}^{\infty} \frac{x \sin mx}{1 + x^2} dx$ (m > 0)

(06 Marks)

c. Find
$$Z_T^{-1} \left[\frac{3z^2 + 2z}{(5z - 1)(5z + 2)} \right]$$
. (06 Marks)

Find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases} \text{ and hence evaluate } \int_0^\infty \frac{\sin^2 t}{t^2} dt.$$
 (08 Marks)

b. Find the Z – transform of 2n + $\sin\left(\frac{n\pi}{4}\right)$ + 1. (06 Marks)

Solve by using Z – transforms $Y_{n+2} - 4$ $Y_n = 0$ given that $Y_0 = 0$, $Y_1 = 2$. (06 Marks) a. Obtain the lines of regression and hence find the coefficient of correlation for the data:

x:	1	3	4	2	5	8	9	10	13	15
y:	8	6	10	8	12	16	16	10	32	32

(08 Marks)

b. Fit a Second degree parabola in the least Square sense for the following data:

x:	1	2	3	4	5
y :	10	12	13	16	19

(06 Marks)

c. Use the Regula-Falsi method to obtain the real root of the equation $\cos x = 3x - 1$ correct to (06 Marks) 3 decimal places in (0, 1).

a. Given the equation of the regression lines x = 19.13 - 0.87y, y = 11.64 - 0.5x. Compute (08 Marks) the mean of x's, mean of y's and the coefficient of correlation.

b. Fit a curve of the form, $y = a e^{bx}$ for the data:

(06 Marks)

Using Newton-Raphson method to find a real root of x $\log \frac{x}{10} = 1.2$ upto 5 decimal places (06 Marks) near x = 2.5.

a. Given $\sin 45^{\circ} = 0.7071$, $\sin 50^{\circ} = 0.7660$, $\sin 55^{\circ} = 0.8192$, $\sin 60^{\circ} = 0.8660$, find $\sin 57^{\circ}$ (08 Marks) using an Backward Interpolation formula.

b. Applying Lagrange's Interpolation formula inversely find x when y = 6 given the data

(06 Marks)

Using Simpson's $\frac{1}{3}$ rule with Seven ordinates to evaluate $\int_{-100}^{8} \frac{dx}{dx}$ (06 Marks)

Fit an Interpolating polynomial for the data $u_{10} = 355$, $u_0 = -5$, $u_8 = -21$, $u_1 = -14$, $u_4 = -125$ 8 by using Newton's Divided difference formula and hence find u2. (08 Marks)

b. Use Lagrange's Interpolation formula to fit a polynomial for the data:

x:	0	Cl.	3	4
у:	-12	0	6	12

Hence estimate y at x =

(06 Marks)

Evaluate $\int \log_e x \, dx$ taking six equal strips by applying Weddle's rule. (06 Marks)

Using Green's theorem, evaluate $\left[\left[(y - \sin x) dx + \cos x \right] dy \right]$, where C is the plane triangle enclosed by the lines y = 0, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. (08 Marks)

b. Using Divergence theorem evaluate $\int \vec{F} \cdot ds$, where $\vec{F} = 4x \, i - 2y^2 \, j + z^2 K$ and S is the surface

bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3.

(06 Marks)

c. Show that the Geodesics on a plane are straight lines.

(06 Marks)

- 10 a. Verify Stoke's theorem for the vector field $\vec{F} = (2x y)i yz^2j y^2z$ K over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. (08 Marks)
 - b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} \frac{d}{dx} \left[\frac{\partial f}{\partial y^1} \right] = 0$. (06 Marks)
 - c. Find the Extremals of the functional

$$\int_{x_0}^{x_1} \frac{y^{1^2}}{x^3} dx.$$

(06 Marks)

Third Semester B.E. Degree Examination, July/August 2021 Electronic Instrumentation

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Define significant figure, accuracy and system error.

(06 Marks)

b. A component manufacturer constructs certain resistors to be anywhere between 1.14 $K\Omega$ and 1.26 $K\Omega$ and classifies them as 1.2 $K\Omega$ resistors. What tolerance should be stated? If the resistance values are specified at 25°C and the resistors have a temperature coefficient of +500 ppm/°C. Calculate the maximum resistance of one of these components at 75°C.

(08 Marks)

c. Mention the requirements of shunt. Explain different types of thermocouples.

(06 Marks)

a. Find the voltage reading and % error of each reading obtained with a voltmeter on
 (i) 5V range (ii) 10V range (iii) 30V range, if the instrument has a 20 KΩ/V sensitivity and
 is connected across R_b of Fig.Q2(a).

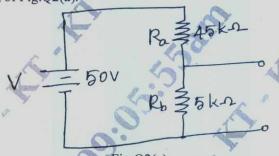


Fig.Q2(a)

(10 Marks)

b. Calculate the value of the multiplier resistor for 10V runs range on the voltmeter shown in Fig.Q2(b).

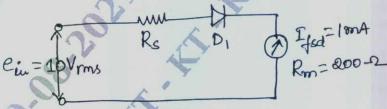


Fig.Q2(b)

(05 Marks)

c. Explain the operation of True rms voltmeter.

(05 Marks)

3 a. Explain successive approximation DVM with suitable diagrams.

(10 Marks)

b. What is the resolution of a 3½ digit display on 1V and 10V ranges?

(05 Marks)

- c. An integrator consists of $100~\text{K}\Omega$ and $2\mu\text{F}$ capacitor. If the applied voltage is 2V, what will be the output of the integrator after 2 seconds. (05 Marks)
- 4 a. Explain with block diagram digital frequency meter.

(10 Marks)

b. Discuss the functioning of Digital Tachometer and Digital Capacitance meter.

(10 Marks)

		1810 Bugged Make, and a sum of a supplied in the land of the land	17EC32
5	a.	Describe the measurement of frequency by Lissajous method.	(08 Marks)
5	b.	Draw Basic CRO block diagram. Explain functionality of each block.	(08 Marks)
	c.	Mention the CRT features.	(04 Marks)
6	a.	Explain function generator with suitable block diagram.	(10 Marks)
	b.	Discuss modern laboratory type signal generator.	(10 Marks)
7	a.	Which is the device/instrument used to measure electrical properties of coils and	d capacitors?
		Explain circuit diagram and its functionality.	(10 Marks)
	b.	Explain field strength meter with suitable circuit diagrams.	(10 Marks)
_			4
8	a.	An unbalanced Wheatstone bridge is given in Fig.Q8(a). Calculate the current	through the
		galvanometer.	
		2 2 2 5 6	
		P. WIK 22 R2=2.5k	
		6V Re-300-22	
		100 76	
		- a - G	
		- 1 2.5k	
		743.5K 5 R4=10K	
		R ₃ 7 7	
		Fig.Q8(a)	(08 Marks)
	b.		(08 Marks)
	c.	A Maxwell bridge is used to measure an inductive impedance. The bridge	
		balance are $C_1 = 0.01 \mu F$, $R_1 = 470 \text{ K}\Omega$, $R_2 = 5.1 \text{ K}\Omega$ and $R_3 = 100 \text{ K}\Omega$. Fin	
		equivalent of the unknown impedance.	(04 Marks)
9	a.	What are the factors to be considered while selecting transducer?	(06 Marks)
	b.	Derive an expression for Gauge factor in bonded resistance wire strain gauge.	(10 Marks)

(04 Marks)

(10 Marks)

(10 Marks)

c. Give advantages and limitations of thermistor.

Explain the construction and working of LVDT.

b. Write a short note on piezo electrical and photo transistor transducers.

(08 Marks)

Third Semester B.E. Degree Examination, July/August 2021 Analog Electronics

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Mention the steps involved for obtaining the AC equivalent of a transistor network.
 - Derive an expressions for input impedance, output impedance and voltage gain for CE fixed bias configuration using re equivalent model.

 (04 Marks)

 (08 Marks)
 - c. Define hybrid parameters and explain hybrid π model with neat sketch.
- 2 a. Draw the circuit diagrams, for transistor re model in common Emitter and common base configuration. (04 Marks)
 - b. Derive expressions for Z_i, Z₀, A_V and A_i for emitter follower configuration using approximate hybrid equivalent model. (08 Marks)
 - c. For the network shown in Fig.Q2(c), without C_E(unbypassed), determine r_e, Z_i, Z₀ and A_v.

Fig.Q2(c)

(08 Marks)

3 a. Mention the differences between JFET and MOSFET.

(04 Marks)

(08 Marks)

- b. Explain with neat sketches operation and characteristics of n-channel enhancement MOSFET. (08 Marks)
- c. Find r_d , Z_i , Z_0 , and A_V for the circuit shown in Fig.Q3(c).

$$\begin{array}{c|c} & 2 \times 1 \\ & & 2 \times 1 \\ & & & \\$$

4 a. Sketch the following circuit diagrams:

i) JFET AC equivalent model of source follower ii) Cascaded FET amplifier. (04 Marks)

b. Derive an expressions for Z_i, Z₀, and A_V using small signal JFET amplifier for self bias configuration (Bypassed Rs). (08 Marks)

c. For the source follower network shown in Fig.Q4(c), determine: i) r_d ii) Z_i iii) Z₀ iv) A_V.

Fig.Q4(c)

$$R_{c_1} = \frac{1}{2} \frac{1}{2}$$

Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8=50, will be treated as malpractice. Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8=50, will be

5 a. An amplifier rated at a 40W output is connected to a 10 Ω speaker find :

i) Input power required for full output if power gain is 25dB

- ii) Input voltage for rated output if the amplifier voltage gain is 40dB. (06 Marks)
- b. Explain high frequency response of JEFT amplifiers.
 c. Explain multistage frequency effects.
- 6 a. Derive an expressions for Miller input and output capacitors. (06 Marks)
 - b. Determine r_e , A_V and R_i for the low frequency response of BJT amplifier circuit shown in Fig.O6(b). Assume $r_0 = \infty$.

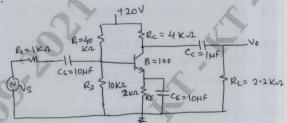
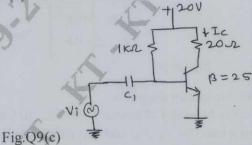


Fig.Q6(b) (08 Marks)

- c. Draw the circuit diagram of:
 - i) High frequency response of BJT amplifier in CE mode with capacitances effects
 - ii) Low frequency response of FET amplifier in common source mode with capacitive elements effects. (06 Marks)
- 7 a. List the conditions for sustained oscillations. (04 Marks)
 - b. Explain with neat circuit diagram, series resonant crystal oscillator using BJT. (08 Marks)
 - c. Design the RC elements of a Wein bridge oscillator for the operation at f = 10KHz and draw the oscillator circuit using op-Amp. (08 Marks)
- 8 a. Explain effect of negative feedback on gain and Bandwidth. (05 Marks)
 - b. Explain with neat circuit diagram, the operation of BJT Colpitt oscillator and mention its advantages over Hartely oscillator. (08 Marks)
 - c. Explain UJT relaxation oscillator with necessary equations and waveforms. (07 Marks)
- 9 a. Classify the power amplifiers and define them with necessary waveforms and 'Q' point.

(06 Marks)

- b. Explain series transistor voltage regulator with neat diagram. (06 Marks)
- c. Calculate input power, output power and efficiency of the series fed class A power amplifier circuit shown in Fig.Q9(c).



(08 Marks)

- 10 a. Define: i) Cross over distortion ii) percentage voltage regulation iii) amplifier efficiency iv) harmonic distortion v) voltage regulator. (10 Marks)
 - b. Explain transformer coupled class A power amplifier with necessary equations. (06 Marks)
 - c. For class 'B' amplifier using a supply of $V_{CC} = 30V$ and driving a load of 16Ω , determine maximum input power and output power. (04 Marks)

CBCS SCHEME

USN

17EC35

Third Semester B.E. Degree Examination, July/August 2021 **Network Analysis**

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

a. Using source transformation techniques, find 'v' for the circuit in Fig.Q1(a).

Fig.Q1(a)

(07 Marks)

b. Obtain equivalent resistance Rab for the circuit in Fig.Q1(b) and hence find 'i'.

Fig.Q1(b)

(07 Marks)

Explain ideal and practical current sources.

(06 Marks)

Determine the current I₀ in the circuit of Fig.Q2(a) using Mesh analysis.

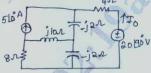


Fig.Q2(a)

(08 Marks)

b. Use nodal analysis to find v_0 in the network of Fig.Q2(b)



Fig.Q2(b)

(08 Marks)

Explain the concept of super node with an illustration.

(04 Marks)

State and prove Reciprocity theorem.

(06 Marks)

b. Use superposition theorem to find i₀ in the circuit shown in Fig.Q3(b).

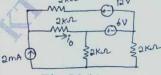


Fig.Q3(b)

(06 Marks)

Find Thevenin's equivalent circuit across the terminals a - b for the circuit shown in Fig.Q3(c). (08 Marks)

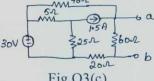


Fig.Q3(c)

1 of 3

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8=50, will be treated as malpractice.

- 4 a. State and prove maximum power transfer theorem for the case of AC source, hence show that $\rho_{max} = \frac{|V_{TH}|^2}{8R_L}$ (08 Marks)
 - b. Find the current through 16 Ω resistor using Norton's theorem in Fig.Q4(b).

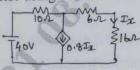
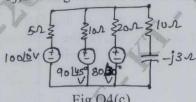


Fig.Q4(b)

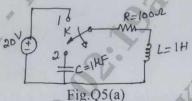
(08 Marks) (04 Marks)

c. Find the current through $(10-3j)\Omega$ using Millman's theorem in Fig.Q4(c).

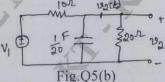


The switch 'K' is changed from position 1 to position 2 at t=0. Steady state condition having been reached at position 1. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$.

[Refer Fig.Q5(a)]



b. In the network shown in Fig.Q5(b), $V_1(t) = e^{-t}$ for $t \ge 0$ and is zero for all t < 0. If the capacitor is initially uncharged. Determine the value of $\frac{d^2v_2}{dt^2}$ and $\frac{d^3v_2}{dt^3}$ at $t = 0^+$.

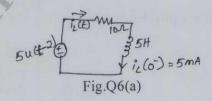


(08 Marks)

c. Explain initial and final conditions in case of a capacitor.

(06 Marks)

- 6 a. For the circuit shown in Fig.Q6(a),
 - (i) Find the differential equation for i_L(t)
 - (ii) Find Laplace transform of i_L(t)
 - (iii) Solve for i_L(t)



(08 Marks)

b. For the circuit shown in Fig.Q6(b), (i) Find the differential equation for i_L(t), (ii) Find (08 Marks) Laplace transform of $i_c(t)$, (iii) Solve for $i_L(t)$.

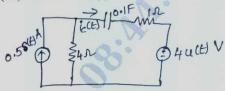


Fig.Q6(b)

Obtain Laplace transform for a decaying exponential signal.

(04 Marks)

- a. Prove that the resonant frequency is the geometric mean of the two half power frequencies (08 Marks) i.e., Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$
 - b. Obtain an expression for quality factor of an capacitor.

(07 Marks)

- c. In a series circuit, $R = 6 \Omega$, $\omega_0 = 4.1 \times 10^6 \text{ rad/sec}$, bandwidth = 10^5 rad/sec . Compute L, C (05 Marks) half power frequencies and Q.
- Obtain an expression for the resonant frequency in a parallel resonant circuit. (08 Marks)
 - b. Show that a two branch parallel resonant circuit is resonant at all frequencies when

 $R_L = R_C = \sqrt{\frac{L}{C}}$

(07 Marks)

c. Find the value of R_L for which the circuit is at resonance, as shown in Fig.Q8(c). (05 Marks)



Obtain an expression for h-parameters in terms of Z-parameters.

(08 Marks)

Find Z and Y parameters for the network shown in Fig.Q9(b).

(08 Marks)

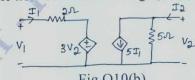
- Explain ABCD parameters.

(04 Marks)

Obtain an expression for Y-parameters in terms of ABCD parameters. 10

(08 Marks)

Find ABCD parameters for the network shown in Fig.Q10(b).



(08 Marks)

- State reciprocity condition for
 - (i) Z parameters
 - (ii) Y parameters
 - (iii) h parameters
 - (iv) ABCD parameters

(04 Marks)

GBGS SCHEME

USN 17EC36

Third Semester B.E. Degree Examination, July/August 2021 Engineering Electromagnetics

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

a. State and explain Coulomb's law in vector form.

(06 Marks)

- b. Point charge $Q_1 = 300 \mu c$ located at (1, -1, 3) experiences a force $F = 8a_x 8a_y 4a_z$ N due to charge Q_2 at (3, -3, 2). Find Q_2 .
- c. Find the total charge within the volume indicated:

i) $\rho_y = 10z^2 e^{-0.1x} \sin \pi y$, $1 \le x \le 2$; $0 \le y \le 1$; $3 \le z \le 3.6$

ii)
$$\rho_{v} = 4xyz^{2}, \quad 0 \le \rho \le 2; \quad 0 \le \phi \le \frac{\pi}{2}; \quad 0 \le z \le 3$$
 (08 Marks)

- 2 a. Derive the expression for electric field intensity 'E' at any point due to uniform line charge of density ρ_{ℓ} c/m. (07 Marks)
 - b. Two uniform surface charge densities of density ρ_s c/m² are located at x = ±4m. Determine the electric field at all the points. (06 Marks)
 - c. Given $D = 5x^2a_x + 10za_z c/m^2$, find the net outward flux for the surface of a cube of 2m on an edge centered at origin. The edges of the cube are parallel to coordinate axes. (07 Marks)
- 3 a. State and prove Gauss law in integral form.

(06 Marks)

- b. Find the numerical value of Divergence of D at the point indicated if:
 - (i) $D = 20xy^2(z+1)a_x + 20x^2y(z+1)a_y + 10x^2y^2a_z$ c/m² at P_A(0.3, 0.4, 0.5)
 - (ii) $D = 4\rho z \sin \phi a_{\rho} + 2\rho z \cos \phi a_{\phi} + 2\rho^2 \sin \phi a_z c/m^2 \text{ at } P_B\left(1, \frac{\pi}{2}, 2\right)$ (06 Marks)
 - c. Given $D = \left(\frac{5r^2}{4}a_r\right) c/m^2$ in spherical coordinates evaluate both sides of divergence theorem

for the volume enclosed between r = 1 m and r = 2 m.

(08 Marks)

- 4 a. Define scalar electric potential. Derive the expression for potential due to a point charge.
 - b. Find the work done in moving a 5 μ c point charge from origin to p(2, -1, 4) through the field $E = 2xyza_x + x^2za_y + x^2ya_z$. V/m via the path:
 - (i) Straight line segments (0, 0, 0) to (2, 0, 0) to (2, -1, 0) to (2, -1, 4)
 - (ii) Straight line x = -2y; z = 2x

(08 Marks)

- c. Given $V = 50x^2yz + 20y^2v$ in free space,
 - (i) Find voltage at P(1, 2, -3)
 - (ii) Field strength E at P.

(06 Marks)

- 5 a. Using Laplace equation derive the expression for capacitance of a co-axial cylindrical capacitor. The boundary conditions are $V = V_0$ at $\rho = a$ and V = 0 at $\rho = b$, b > a. (10 Marks)
 - b. In spherical coordinates V = 865 V at r = 50 cm and E = 748.2 a_r V/m at r = 85 cm. Determine the location of voltage reference if the potential depends only on 'r'. (10 Marks)

- 6 a. State and explain Biot-Savart's law. (05 Marks)
 - b. Find 'H' at origin due to an infinite conductor carrying a current of 5A in a_y direction and located at x = 2 and z = -2. (07 Marks)
 - c. Given $H = \frac{x + 2y}{z^2} a_y + \frac{2}{z} a_z$ A/m, find J. Find total current passing through z = 4; $1 \le x \le 2$; $3 \le y \le 5$. (08 Marks)
- 7 a. The point charge Q = 18 nc has a velocity of 5×10^6 m/s in the direction $a_v = 0.60a_x + 0.75a_y + 0.30a_z$. Calculate the magnitude of force exerted on the charge by: (i) $B = -3a_x + 4a_y + 6a_z$ mT (ii) $E = -3a_x + 4a_y + 6a_z$ KV/m (06 Marks)
 - b. Derive the expression for the force on a differential current element moving through a steady magnetic field. (08 Marks)
 - c. The field $B = -2a_x + 3a_y + 4a_z$ mT is present in free space. Find vector force exerted on a straight wire carrying 12 A in a_{AB} direction, given A(1, 1, 1) and (i) B(2, 1, 1) (ii) B(3, 5, 6). (06 Marks)
- 8 a. Define Magnetization. Given a ferrite material which is operating in a linear mode with B=0.05~T and $\mu_T=50$. Calculate χ_m , M and H. (06 Marks)
 - b. Derive the boundary conditions for magnetic fields B, H and M for the interface between the different magnetic media. (07 Marks)
 - c. Let $\mu_1 = 4 \mu H/m$ in region 1 where z > 0 while $\mu_2 = 7 \mu H/m$ in region 2 where z < 0, $K = 80 a_x A/m$ on the surface z = 0. If $B_1 = 2a_x 3a_y + a_z mT$ in region 1, find B_2 . (07 Marks)
- 9 a. An area of 0.65 m² in $z \neq 0$ plane is enclosed by a filamentary conductor. Find the induced voltage given $B = 0.05 \cos 10^3 t \left[\frac{a_y + a_z}{\sqrt{2}} \right] T$. (06 Marks)
 - b. What is inconsistency of Ampere's law with continuity equation? How it was modified by Maxwell? Derive the modified equation. (06 Marks)
 - c. Given $E = E_m \sin(\omega t \beta z)a_y V/m$ in free space, find D, B, H. Sketch E and H at t = 0.

 (08 Marks)
- 10 a. Prove that the intrinsic impedance of a perfect dielectric $\eta = \frac{|E|}{|H|} = \sqrt{\frac{\mu}{\epsilon}}$ (06 Marks)
 - b. Derive expressions for attenuation constant 'α' and phase constant 'β' for any conducting media. (06 Marks)
 - c. Calculate attenuation constant, wave velocity and intrinsic impedance in sea water for a uniform plane wave at 10 GHz. The constants are $E_r = 80$, $\mu_r = 1$, $\sigma = 4$ Mho s/m. (08 Marks)

Third Semester B.E. Degree Examination, July/August 2021
Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Find the modulus and amplitude of $\frac{4+2i}{2-3i}$.

(06 Marks)

b. Find a unit vector normal to both the vectors 4i - j + 3k and -2i + j - 2k. Find also sine of the angle between them. (07 Marks)

c. Show that $\begin{bmatrix} \overrightarrow{a} + \overrightarrow{b}, \overrightarrow{b} + \overrightarrow{c}, \overrightarrow{c} + \overrightarrow{a} \end{bmatrix} = 2 \begin{bmatrix} \overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c} \end{bmatrix}$

(07 Marks)

2 a. Express $(2+3i)+\frac{1}{1-i}$ in x+iy form.

(06 Marks)

b. Find the modulus and amplitude of $1 + \cos \theta + i \sin \theta$.

(07 Marks)

c. Find λ so that $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{j} + \lambda \hat{k}$ are coplanar.

(07 Marks)

3 a. Find the nth derivative of $e^{ax} \cos(bx + c)$.

(06 Marks)

b. Find the angle of intersection of the curves $r = \sin \theta + \cos \theta$ and $r = 2 \sin \theta$.

(07 Marks)

c. If, z = f(x, y) where $x = e^{u} + e^{-v}$, $y = e^{-u} - e^{v}$. Prove that $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$.

(07 Marks)

4 a. If $y = \tan^{-1} x$, then show that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$.

(06 Marks)

b. Find the pedal equation for the curve $\frac{2a}{r} = 1 + \cos \theta$.

(07 Marks)

(07 Marks)

c. If, $u = x^2 + y^2 + z^2$, v = xy + yz + zx, w = x + y + z, then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$

(06 Marks)

a. Obtain the reduction formula for $\int \cos^n x dx$. b. Using reduction formula, find the value of $\int_0^1 x^2 (1-x^2)^{\frac{3}{2}} dx$.

(07 Marks)

c. Evaluate $\iint_{-\infty}^{1/2} \int_{-\infty}^{x+z} (x+y+z) dx dy dz$.

(07 Marks)

2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice. Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages

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- a. Evaluate $\int x \sin^8 x \, dx$. (06 Marks)
 - b. Evaluate $\int_{-\infty}^{1} \int_{-\infty}^{\sqrt{1-y^2}} x^3 y \, dx \, dy.$ (07 Marks)
 - c. Evaluate $\int x \sin^2 x \cos^4 x dx$. (07 Marks)
- A particle moves along the curve $\vec{r} = 3t^2\hat{i} + (t^3 4t)\hat{j} + (3t + 4)\hat{k}$. Find the component of velocity and acceleration at t = 2 in the direction of $\hat{i} - 2\hat{j} + 2\hat{k}$.
 - b. Find the angle between the tangents to the surface $x^2y^2 = z^4$ at (1, 1, 1) and (3, 3, -3).
 - Find div \vec{F} and curl \vec{F} where $\vec{F} = \nabla(x^3 + y^3 + z^3 3xyz)$ (07 Marks)
- Find the angle between the tangents and to the curve $\vec{r} = \left(t \frac{t^3}{3}\right)\hat{i} + t^2\hat{j} + \left(t + \frac{t^3}{3}\right)\hat{k}$ (06 Marks)
 - Find the directional derivative of $f = x^2yz + 4xz^2$ at (1,-2,-1) along $2\hat{i} \hat{j} 2\hat{k}$. (07 Marks)
 - Prove that $\operatorname{div}(\operatorname{curl} \vec{F}) = 0$. (07 Marks)
- a. Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{42y}$. (06 Marks)
 - (07 Marks)
 - b. Solve $x^2ydx (x^3 + y^3)dy = 0$. c. Solve $\frac{dy}{dx} \frac{2y}{x} = x + x^2$. (07 Marks)
- 10 a. Solve $xdy ydx = \sqrt{x^2 + y^2}dx$. (06 Marks)
 - b. Solve $(5x^4 + 3x^2y^2 2xy^3)dx + (2x^3y 3x^2y^2 5y^4)dy = 0$. (07 Marks)
 - c. Solve $\frac{dy}{dx} \frac{y}{x+1} = e^{3x}(x+1)$. (07 Marks)